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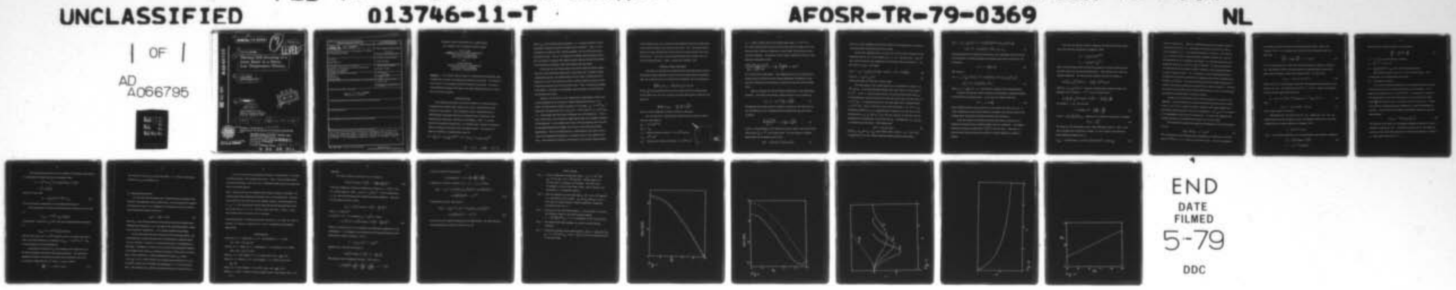

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Thermal Self-Focusing of a Laser Beam in a Dense, Low Temperature Plasma.

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ANDREW SCHMITT

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THERMAL SELF-FOCUSING OF A LASER BEAM IN A DENSE, LOW TEMPERATURE PLASMA

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Abstract - The thermal self-focusing of a Gaussian electromagnetic beam incident upon a dense, low temperature plasma is discussed. The behavior of the beam as it propagates through the plasma is analyzed. A measure of the penetration depth of the beam in an inhomogeneous plasma as a function of its power is given.

INTRODUCTION

The nonuniform intensity of the electric field in a Gaussian electromagnetic beam incident upon a plasma can cause a redistribution of the electrons in the plasma. This mechanism leads to the phenomenon of self-focusing and enables the beam to penetrate into an otherwise overdense plasma (Akhmanov, Sukhorukov and Khoklov, 1968). It is well-known from geometric optics that a ray incident upon a region of increasing index of refraction will refract into the higher region. In a plasma the index of refraction is approximately given by

$$[1 - (\omega_{pe}^2/\omega^2) - i(\nu_{ei}\omega_{pe}^2/\omega^3)]^{1/2} = [(1 - (n_e/n_{ec}) - i(\nu_{ei} n_e/\omega n_{ec})]^{1/2}$$

where n_{ec} is the critical electron density, i.e. the plasma density where the plasma frequency equals the incident laser frequency. Thus, if a ray is incident on a plasma with a density gradient, the ray will refract into a less dense region. Hence, if somehow the incident electromagnetic radiation can induce a change in the initially uniform plasma density distribution in the transverse direction, then self-focusing may take place.

In the case of a fast (laser pulse time shorter than the energy relaxation time of the electrons) pulsed laser interacting with a strongly ionized collisionless plasma, the ponderomotive force is able to redistribute the electrons in such a way that there exists a positive density gradient in the transverse direction away from the beam axis. Thus a transverse gradient of the index of refraction is established, and this leads to self-focusing of the incident laser beam.

However, in the case of a low temperature, high density plasma such as that employed in our laboratory Z-pinch experiment (Rockett, et al., 1978), the electron-ion collision frequency is very high, $\nu_{ei} \simeq 3 \times 10^{12}$ sec.$^{-1}$ Moreover, the pulse time of our CO_2 laser is of the order of 3.5×10^{-8} sec., i.e., much longer than the energy relaxation time of the electrons. Therefore, in this case, the nonlinearity appearing through the nonuniform heating of the electrons by the incident laser beam is more important than that due to the ponderomotive force effect, and we have a thermal self-focusing phenomenon. This is characterized by a redistribution of the plasma electrons by the nonuniform heating in a direction normal to the incident beam axis. The transverse nonuniform distribution of the electric field intensity

of the incident laser beam creates a nonhomogeneous electron temperature in the radial direction away from the beam axis. For a Gaussian electric field in the laser the electrons on the beam axis become hotter than those away from it. This also can cause self-focusing of the incident beam as will be discussed below. (Sodha, Ghatak and Tripathi, 1976).

THERMAL SELF-FOCUSING

The energy of the electrons acquired from the incident laser beam is dissipated through collisions with the ions and through thermal conduction. The rate of energy lost due to electron-ion collisions may be described by

$$\frac{d}{dt}\left(\frac{3}{2}k_B T_e\right)_{e-i} = \frac{3}{2}k_B\left(2m\nu_{eq}(T_e - T_i)/M\right)$$

where ν_{eq}^{-1} is the equilibration time of the electron and ion temperatures. On the other hand, the rate of energy loss per electron due to thermal conduction is

$$\frac{d}{dt}\left(\frac{3}{2}k_B T_e\right)_{tc} = -\frac{1}{n_e r}\frac{\partial}{\partial r}\left(r\kappa\frac{\partial T_e}{\partial r}\right)$$

where κ is the coefficient of electron thermal conduction.

For our laboratory Z-pinch plasma experiment the ambient conductions are described by

$T_{eo} = 2.32 \times 10^5\ {}^oK.$

$T_{io} \lesssim T_{eo}$

V_e = electron thermal velocity $\simeq 4.2 \times 10^8$ cm/sec.

$\nu_{eq} \simeq 10^{10}\ \mathrm{sec}^{-1}$

ν_{ei} = electron-ion collision frequency $\simeq 3 \times 10^{12}\ \mathrm{sec}^{-1}$

r_o = radius of focal spot of the incident laser beam $\approx 6 \times 10^{-3}$ cm.

For these experimental values it may be shown that the energy loss of the electrons due to thermal conduction is much larger than that due to electron-ion collisions. An estimate of the relative importance of the two dissipation mechanisms yields

$$\frac{(3/2)k_B\, 2m\nu_{eq}(T_e - T_i)/M}{\frac{\partial}{\partial r}(r\,\kappa\, \frac{\partial Te}{\partial r})/n_e r} \cong 3\,\frac{m}{M}\,\frac{r_o^2\,\nu_{eq}\,\nu_{ei}}{V_e^2} \cong O(10^{-2})$$

for our laboratory experiment. This suggests that we have a thermal focusing process in which the main energy dissipation mechanism is electron heat conduction in the direction transverse to the axis of the incident beam.

Next we calculate the electron density distribution in the transverse direction. The steady state energy balance equation may be written as

$$e\,\underset{\sim}{E} \cdot \underset{\sim}{V} = (n_e\, r)^{-1}\frac{d}{dr}(r\,\kappa\,\frac{d\,T_e}{dr}) \qquad (1)$$

Substituting the velocity from the equation of motion for the electrons under the influence of the oscillating electric field of the CO_2 laser beam, we obtain

$$\frac{n_e\,\nu_{ei}\,e^2\,EE^*}{m\,\omega^2} \simeq -\frac{1}{r}\,\frac{d}{dr}(r\,\kappa\,\frac{d\,T_e}{dr}) \qquad (2)$$

where ω is the frequency of the incident CO_2 laser beam, and we have used the fact that $\nu_{ei}^2 << \omega^2$ in our experiment. We now assume a Gaussian distribution for the incident laser beam:

$$EE^* = E_o^2\,\exp[-\,r^2/r_o^2 f^2(z)]/f^2(z) \qquad (3)$$

where E_o is the amplitude of the electric field of the incident beam, and f(z) is a variable which parametrizes the beam width.

In the linear approximation we carry out a Taylor series expansion of the right hand side of eq. (3) with respect to r = 0, i.e., the beam axis. Also we use ambient values for the coefficients in the energy balance equation (2). We then solve for $T_e(r)$, and obtain

$$T_e(r) = T_{eo} + (\beta_o E_o^2/4\kappa_o f^2)\{r_o^2[1 - (1/4f^2)] - r^2[1 - (r^2 4r_o^2 f^2)]\} \tag{4}$$

$$\text{where } \beta_o \equiv n_{eo} e^2 \nu_{eio}/m\omega^2 \tag{5}$$

and the subscript zero denotes ambient values.

The sound speed in the z-pinch plasma in our experiment is about 2.2×10^6 cm/sec. As the characteristic length of the laser beam is of the order of 6×10^{-3} cm (i.e., the focal spot radius), the characteristic time scale t_H for hydrodynamic motions transverse to the laser beam axis is of the order of 3×10^{-9} sec. As indicated previously the laser pulse time is approximately 3.5×10^{-8} sec. Thus, in the time scale of the laser pulse, i.e., for $t > t_H$, the plasma pressure $p_e + p_i$ is approximately constant across the laser beam. Furthermore, $n_e \cong n_i$, and $T_e \cong T_i$ as the time scale for the electron and ion temperatures to equilibrate is much less than t_H. Consequently, $p_e = n_e k_B T_e \cong$ constant across the beam. This implies that

$$n_e(r) \cong n_{eo} T_{eo}/T_e(r) \tag{6}$$

where $n_e = n_{eo}$ and $T_e = T_{eo}$, the ambient values, when $r = r_o$. From $T_e(r)$ as expressed in eq. (4) we then obtain the electron density distribution:

$$n_e(r) = n_{eo} - n_{eo}[\{(r_o^2 - r^2) - (1/4r_o^2f^2)(r_o^4 - r^4)\}/\{(4\kappa_o T_{eo} f^2/\beta_o E_o^2)$$

$$+ [(r_o^2 - r^2) - (1/4r_o^2f^2)(r_o^4 - r^4)]\}]; \quad 0 \leq r \leq r_o \tag{7}$$

This expression for the density distribution is now substituted into the formula for the dielectric constant (neglect the imaginary part which accounts for the absorption)

$$\epsilon(r) = 1 - [\omega_p^2(r)/\omega^2] \tag{8}$$

The result is

$$\epsilon(r) = \epsilon_o + (\omega_{po}^2/\omega^2)(\{(r_o^2 - r^2) - (1/4r_o^2f^2)(r_o^4 - r^4)\}/\{(4\kappa_o T_{eo} f^2/\beta_o E_o^2)$$

$$+ [(r_o^2 - r^2) - (1/4r_o^2f^2)(r_o^4 - r^4)]\}) \tag{9}$$

where $\epsilon_o = 1 - (\omega_{po}^2/\omega^2)$, i.e., the dielectic constant of the ambient plasma.

Equation (9) gives the radial variation of the dielectric constant necessary for the thermal focusing process. It may also be written in the form

$$\epsilon(r) = \epsilon_o + \phi(E_o^2) \tag{10}$$

where $\phi(E_o^2)$ is given by the second term of the right hand side of eq. (9). It is the modification of the plasma dielectric constant by the electric field of the incident beam through the nonuniform heating of the electrons.

In the last section we solved the plasma heat balance equation by expanding the Gaussian beam intensity profile about its centerline. This approximation is strictly not valid when $r^2/r_o^2f^2$ is of order unity. Therefore we solve the heat balance equation numerically by finite difference method as follows.

First we note that the collision frequency and electron thermal conductivity vary with the temperature as (Spitzer, 1956)

$$\nu_{ei} = \nu_{eio}[T_{eo}/T_e(r)]^{3/2}$$

$$\kappa_e = \kappa_{eo}[T_e(r)/T_{eo}]^{5/2}$$

where we have assumed that the electron pressure is nearly constant across the laser beam (see the previous discussion). The subscript zero indicates ambient conditions. The heat balance equation may now be written as

$$(\beta_o T_{eo}^6/\kappa_{eo} T_e^{7/2}) EE^*(r) = -\frac{1}{r}\frac{\partial}{\partial r}(r\, T_e^{5/2}\partial T_e/\partial r)$$

where $\beta_o = n_o \nu_{eio} e^2/m\omega^2$. Using the Gaussian beam intensity profile and defining a new variable, $F = T^{7/2}$, we can rewrite this as

$$(\beta_o T_{eo}^6 E_o^2/\kappa_{eo} f^2) F(r) \exp(-r^2/r_o^2 f^2) = -\frac{2}{7}\frac{1}{r}\frac{d}{dr}(r\frac{dF}{dr})$$

By letting $R = r/r_o f$, this becomes

$$-\lambda\, F(R)\exp(-R^2) = \frac{1}{R}\frac{dF}{dR} + \frac{d^2F}{dr^2} \tag{11}$$

where $\lambda = \frac{7}{2}\beta_o T_{eo}^6 E_o^2 r_o^2/\kappa_{eo}$. Equation (7) is subject to the boundary condition

$$F(r_o) = T_{eo}^{7/2} \tag{12}$$

We solve eq. (11) numerically using a finite difference scheme. The R variation is divided into N intervals of length ΔR; this results in the following finite difference equation for F:

$$F_{n+1} = \{1/2n+1)\}\{4nF_n - n(\Delta R)^2 \lambda F_n^{-1}\exp(-n^2\Delta R^2) - (2n-1)F_{n-1}\} \tag{13}$$

First we estimate F_o. Then F_1 is obtained using the heat balance equation above (valid for $\Delta R << 1$). With these two starting values for F_o and F_1, the succeeding F_n $(n > 1)$ is generated using eq. (13) above. The value of F_n so obtained is then compared with the boundary condition (12), and the next estimate for F_o is adjusted accordingly. An iteration process is now employed until F_N is as close to $T_{eo}^{7/2}$ as desired. The electron temperature distribution in the radial direction obtained by this numerical scheme is given in figures 1 and 2. As a comparison, the corresponding electron temperature distribution calculated by expanding the Gaussian incident beam profile is also shown. Note that the linearized method yields unphysical results for the electron temperature distribution as shown in figure 2. However, for values of the incident beam electric field intensity not far above threshold, the linearized method yields reasonable results.

Another way of solving the energy balance equation (2) is given in the Appendix. This procedure is an improvement upon the linearization method. In fact, it is an exact method of solution except for the initial integration of the energy balance equation, where the coefficient $\beta = ne^2 \nu_{ei}/m\omega^2$ is assumed to be independent of the electron temperature. As seen in the Appendix this method also leads to an expression similar to eq. (9) for $\phi(E_o^2)$.

The fundamental equation governing the propagation of an electromagnetic wave is

$$\nabla^2 \underset{\sim}{E} - \nabla(\nabla \cdot \underset{\sim}{E}) = \omega^2 \epsilon \underset{\sim}{E}/C^2 \qquad (13)$$

The second term on the left hand side is negligible in comparison to the first term if $k^{-2}\nabla^2 (\ln \epsilon) << 1$, where k is the wave vector. The above inequality

is satisfied in almost all cases of practical interest (Sodha, Ghatak and Tripathi, 1976). For a cylindrically symmetric electromagnetic beam we then have

$$\frac{\partial^2 \underset{\sim}{E}}{\partial z^2} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \underset{\sim}{E}}{\partial r}\right) = \omega^2 \epsilon \underset{\sim}{E}/C^2 \tag{14}$$

To solve this equation we follow the method employed by Akhmanov, Sukhorukov, and Khokhlov (1968) using the paraxial ray approximation in geometric optics (see also Sodha, Ghatak and Tripathi, 1976). We first write

$$\epsilon(r, z) = \epsilon(r=0, z) + \phi(< E^2 >) \tag{15}$$

where $\phi(< E^2 >)$ is the dielectric response to the incident electromagnetic beam given by eq. (12). We assume that $\phi(< E^2 >) << \epsilon(r=0)$. Using a WKB approximation we may express $\underset{\sim}{E}$ in the form:

$$\underset{\sim}{E} = \underset{\sim}{A}(r, z)[\epsilon^{1/4}(0)/\epsilon^{1/4}(z)]\exp\{i(\omega t - \int^z dz_1 \omega\, \epsilon^{1/2}\, c)\} \tag{16}$$

with

$$\underset{\sim}{A}(r, z) = \underset{\sim}{A}_o(r, z)\exp[-i\,S(r, z)] \tag{17}$$

and S is the eikonal of the wave.

Substituting eqs. (15) and (16) into eq. (13), making use of eq. (14), and separating the real and imaginary parts, the following solution for a Gaussian incident beam is obtained (Sodha and Tripathi, 1977):

$$S = \frac{1}{2}\gamma(z)r^2 + \phi_1(z)$$

$\phi_1(z)$ = an arbitrary function of z to be determined by the boundary conditions

$$A_o^2 = (E_o^2/f^2)\exp(-r^2/r_o^2 f^2) \tag{18}$$

$$\gamma = (\omega\, \epsilon^{1/2}/c)\,\frac{1}{f}\, df/dz$$

where the beam width parameter f is governed by

$$\frac{d^2 f}{d\xi^2} = \frac{1}{f^3} - \frac{K}{f} - a\,\frac{df}{d\xi} \tag{19}$$

$\xi = z/k\,r_o^2$ is a dimensionaless "distance"

$a = (r_o^2\, c\, \epsilon^{1/2}/2\omega)\, d\epsilon/dz$

$K \equiv \omega_{po}^2 E_o^2 \beta\, r_o^4 / 4c^2 \kappa\, T_{eo}$ is the "self-focusing constant."

The initial conditions on f at $\xi = 0$ are f=1 and $df/d\xi = 0$. This corresponds to an incident plane wave front.

(a) Homogeneous plasma

Let us first consider propagation in a homogeneous plasma; in this case the last term on the right hand side of eq. (19) vanishes. For a plane wave incident at $\xi = 0$ we have $d^2 f/d\xi^2 = 0$ everywhere, if the first and second terms on the right hand side of eq. (19) cancel each other. Thus, $\frac{df}{d\xi}$ will remain at zero, and hence f=1 for all values of ξ. In this case the beam propagates without modification, neither diffracting nor focusing. This condition thus yields the threshold intensity for thermal self-focusing:

$$E_{o\,crit}^2 = 4c^2 k_B T_{eo}/\omega_{po}^2 \beta\, r_o^4 \tag{20}$$

The behavior of the beam as it propagates through the plasma (2) may be obtained by solving eq. (19). A numerical analysis of eq. (19) yields the variation of the beam width parameter f with the dimensionless "distance" ξ. The result is shown in fig. 3.

The self-focusing constant K can be related to the incident beam power by integrating the expression for the Poynting vector

$$P = (\epsilon_o^{1/2} c/8\pi) \int_0^{\infty} dr\, 2\pi r (E_o^2/f^2) \exp(- r^2/r_o^2 f^2)$$

$$= \epsilon_o^{1/2} c\, E_o^2 r_o^2/8$$

Hence, we may write

$$K = 2\omega_{po}^2 \beta r_o^2 P/c^3 \epsilon_o^{1/2} \kappa T_{eo} \qquad (21)$$

Thus K is directly proportional to the incident laser power P.

The critical power for thermal self-focusing is obtained from eq. (20) as

$$P_{crit} = c^3 \epsilon_o^{1/2} \kappa T_{eo}/2\omega_{po}^2 \beta r_o^2$$

Using Spitzer's values for ν_{ei} and K this can be approximately expressed as

$$P_{crit} \cong 4.3 \times 10^{39}\, T_e^5/n_e^3 \text{ ergs/sec.}$$

With the laser power of 3.7×10^{14} ergs/sec used in our laboratory experiment, the critical density is calculated to be $n_{ec} = 2 \times 10^{17}$ cm^{-3}. This is well satisfied by our Z-pinch plasma.

A parameter of interest is f_m, the minimum value attained by f as the beam propagates through the homogeneous plasma. This parameter signifies the extent of focusing associated with a particular value of K. It is found by integrating eq. (19) with $\alpha = 0$ once to obtain

$$\left(\frac{df}{d\xi}\right)^2 = 1 - (1/f^2) - 2K \ln f \qquad (22)$$

The minimum value of f_m occurs when $df/d\xi = 0$. Figure 4 shows the behavior of f_m as a function of K.

(b) Inhomogeneous plasma.

For the case when the plasma has a positive density gradient in the direction of propagation of the incident laser, we can calculate the penetration distance of beam. For simplicity we consider the case of a linearly increasing density profile of the form

$$n_{eo}(z) = \hat{n}_e[1 + (z/L)] \tag{23}$$

where $\hat{n}_e$ is the electron density at the position where the incident laser impinges upon the plasma, i.e., the edge of the cylindrical plasma column in our laboratory experiment. L is a density gradient scale length.

As the beam penetrates into the plasma with increasing density, the axial dielectric constant decreases and a turning point is expected where $\epsilon(r=0, z)$ vanishes. However, our method of analysis is not applicable at this point. Therefore, in order to have an idea of the penetration distance of an incident laser beam as a function of the power of the beam, we introduce, albeit arbitrarily, a typical penetration distance, z_p, where $\epsilon(r=0, z_p) = 0.69$. At this position, the ambient plasma density at $r=r_o$ is, of course, larger than the density corresponding to $\epsilon = 0.69$ at the beam axis. The variation of z_p with the incident beam power is shown in fig. 5.

It is to be noted that the penetration distance is proportional to the electric field strength of the incident laser beam. Thus a beam of higher power will penetrate deeper, and in this way a sufficiently intense beam can penetrate into an overdense plasma.

Note - Recently Sharma and Tripathi (1978) made an attempt to interpret the experimental results obtained by Rockett, et al. in our laboratory. However, they assumed an incorrect laser beam profile, namely, one which peaked at $r = r_o > 0$. Our observations clearly indicates that the focussed laser beam has a maximum at $r = 0$, i.e., the axis of the laser beam. Hence, a Gaussian profile is the more correct one to assume.

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REFERENCES

Akhmanov, S. A., Sukhorukov, A. P., and Khokhlov, R. V. (1968) Sov. Phys. Usp. 10, 609.

Rockett, P. D., Steel, D. G., Ackenhusen, J. G. and Bach, D. R. (1978), Phys. Rev. Lett. 40, 649.

Sharma, J. C. and Tripathi, V. K., (1978), Phys. Rev., A18, 1381.

Sodha, M. S., Ghatak, A. K., and Tripathi, V. K. (1976), Prog. Opt. 13, 169.

Sodha, M. S. and Tripathi, V. K. (1977), Phys. Rev. A16, 2101.

Spitzer, L. (1965), "Physics of Fully Ionized Gases," Interscience Publ., Inc. N. Y.

Appendix -

The energy balance equation (2) may be written as

$$(7\beta E_o^2/2\theta f^2)\exp(-r^2/r_o^2f^2) = -\frac{1}{r}\frac{d}{dr}\{r\frac{d}{dr}(T_e^{7/2})\} \qquad (A1)$$

where the coefficient of thermal conduction is written as $\kappa = \theta T_e^{5/2}$, with θ a constant (Spitzer, 1965). Also $\beta \equiv n\nu_{ei}e^2m\omega^2$. We now assume that $\beta = \beta_o$, where the subscript zero indicates ambient conditions. Then (A1) may be integrated once to yield

$$(c_1/r) + (\rho r_o^2f^2/2r)\exp(-r^2/r_o^2f^2) = \frac{d}{dr}(T_e^{7/2})$$

where $\rho \equiv 7\beta E_o^2/2\theta f^2$.

As $dT_e/dr = 0$ at $r = 0$, therefore $c_1 = -\rho r_o^2f^2/2$. Hence

$$-(\rho r_o^2f^2/2r) + (\rho r_o^2f^2/2r)\exp(-r^2/r_o^2f^2) = \frac{d}{dr}(T_e^{7/2}) \qquad (A2)$$

Before we integrate (A2) let us consider the temperature dependence of the coefficient β. As β depends on the temperature through the density and the electron-ion collision frequency, we let

$$\beta = \beta_o(T_o/T_e)^{7/2} = \beta_1T_e^{-7/2}.$$

Equation (A2) may then be written as

$$(\beta_1r_o^2f^2/r)\{\exp(-r^2/r_o^2f^2) - 1\} = \frac{d}{dr}(T_e^7)$$

This equation can be integrated formally. The result is

$$c_2 + \frac{1}{2}\beta_1r_o^2f^2\{-\frac{r^2}{r_o^2f^2} + \frac{r^4}{4r_o^4f^4} - \frac{r^6}{18r_o^6f^6} + \ldots\} = T_e^7(r)$$

where we have used the formula

$$\int (1/x)\exp(ax)dx = \ln x + \frac{ax}{1!} + \frac{a^2x^2}{2.\,2!} + \frac{a^3x^3}{3.\,3!} + \dots$$

Applying the boundary condition $T_e\,(r = r_o) = T_{eo}$, we obtain

$$T_e(r) = T_{eo}\{1 + (7\beta_o E_o^2/4\kappa_o T_{eo} f^2)[(r_o^2 - r^2) - (1/4r_o^2 f^2)(r_o^4 - r^4 + \\ + (1/18 r_o^4 f^4)(r_o^6 - r^6) - \dots]\}^{1/7} \qquad \text{(A3)}$$

Proceeding as before this leads to

$$\phi(E_o^2) = (\omega_{po}^2/\omega^2)\{1 - [1/\{1 + (7\beta_o E_o^2/4\kappa_o T_{eo} f^2)[(r_o^2 - r^2) \\ - (1/4r_o^2 f^2)(r_o^4 - r^4) + \dots]\}^{1/7})\} \qquad \text{(A4)}$$

If we carry out a binomial expansion of the denominator, we obtain the previous expression for $\phi(E_o^2)$ as shown in eq. (9).

Figure Captions

Fig. 1 Electron temperature versus beam radius. $n_{eo} = 2 \times 10^{17}/cm^3$; $T_{eo} = 20$ eV; $E_o^2 = 2.66 \times 10^9$ ergs/cm^3; Incident beam is at critical power for thermal self-focusing. The solid curve corresponds to results from linear theory, and the dotted curve corresponds to the numerical results.

Fig. 2 Electron temperature versus beam radius. $E_o^2 = 4.26 \times 10^9$ ergs/cm^3, i.e., beam above critical power. n_{eo} and T_{eo} same as in fig. 1. Note that the linear analysis yields an unphysical temperature distribution near $r/r_o = 1$.

Fig. 3 Variation of the beam width parameter f with distance into plasma for different values of the self-focusing strength $K = \beta r_o^4 \omega_p^2 R_o^2/4C^2 \kappa T_{eo}$. $K = 1.0$ corresponds to the critical power.

Fig. 4 The minimum beam width parameter f_m versus the self-focusing strength K.

Fig. 5 Penetration distance versus beam intensity, where $\alpha = \omega p_o^2 \beta_o r_o^4/4C^2 \kappa T_{eo}$. $n_{eo} = 2 \times 10^{17}/cm^3$; $T_{eo} = 20$ eV. αE_o^2 is the self-focusing strength K of the laser beam.

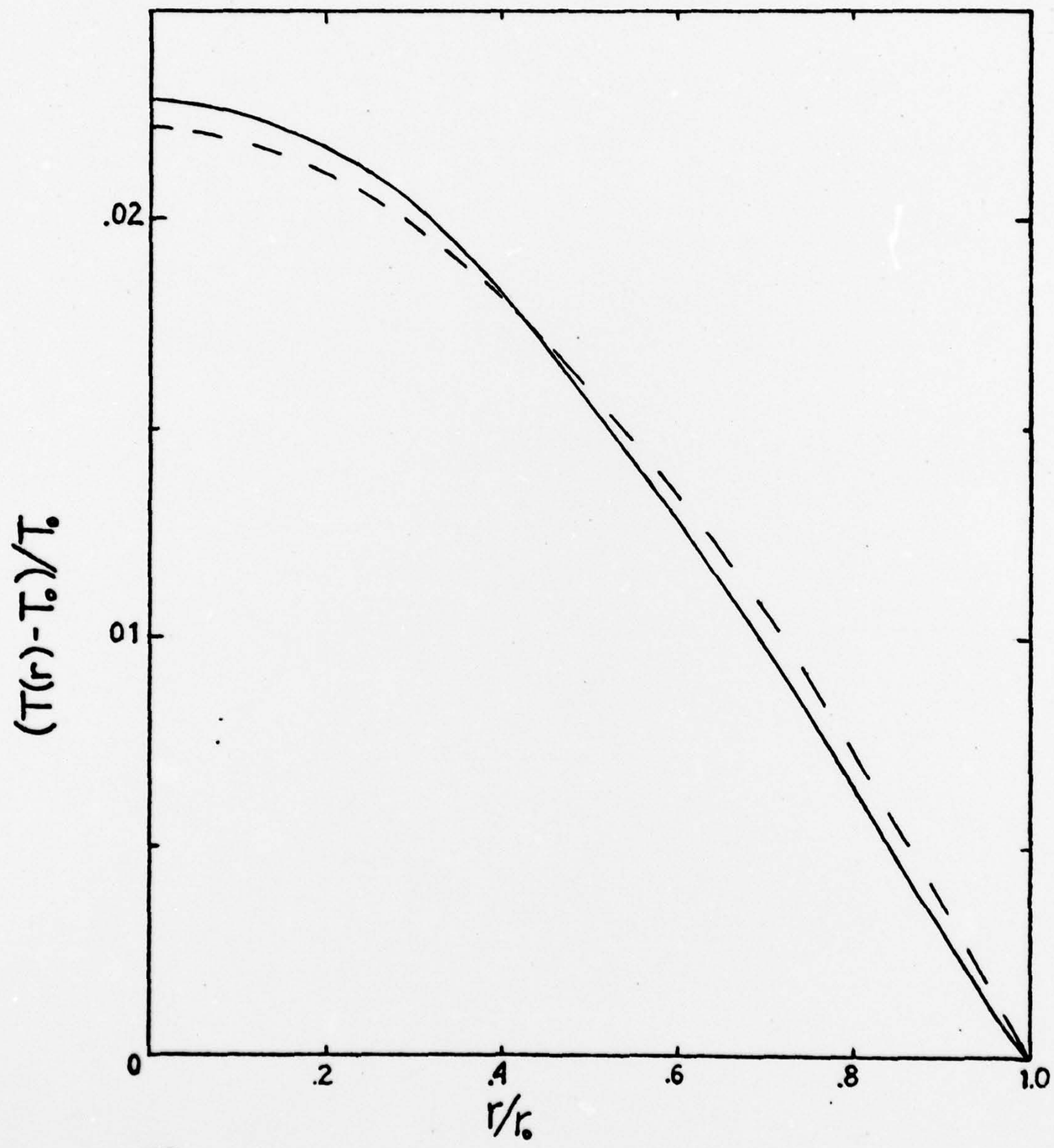

Fig. 1

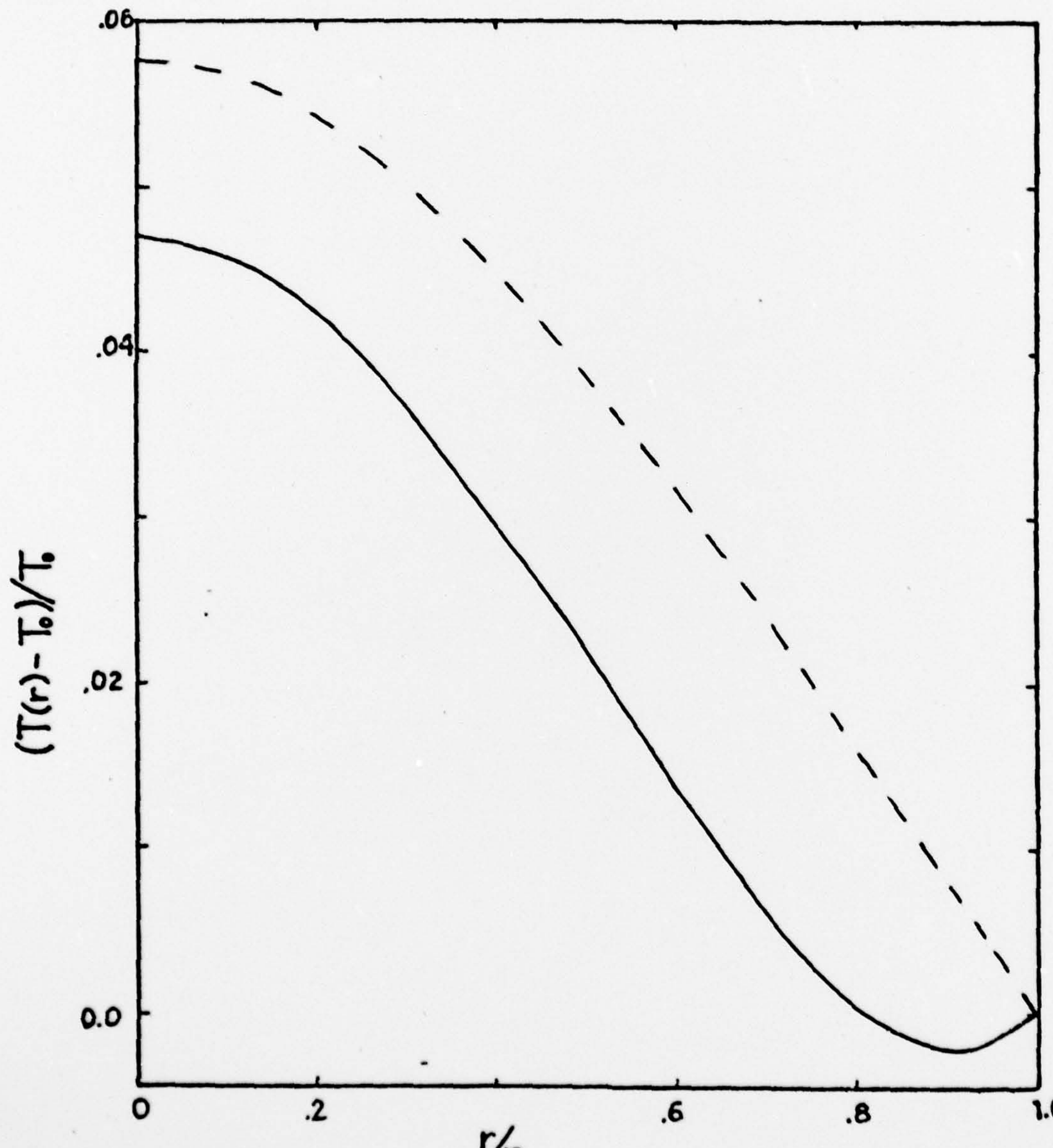

Fig. 2

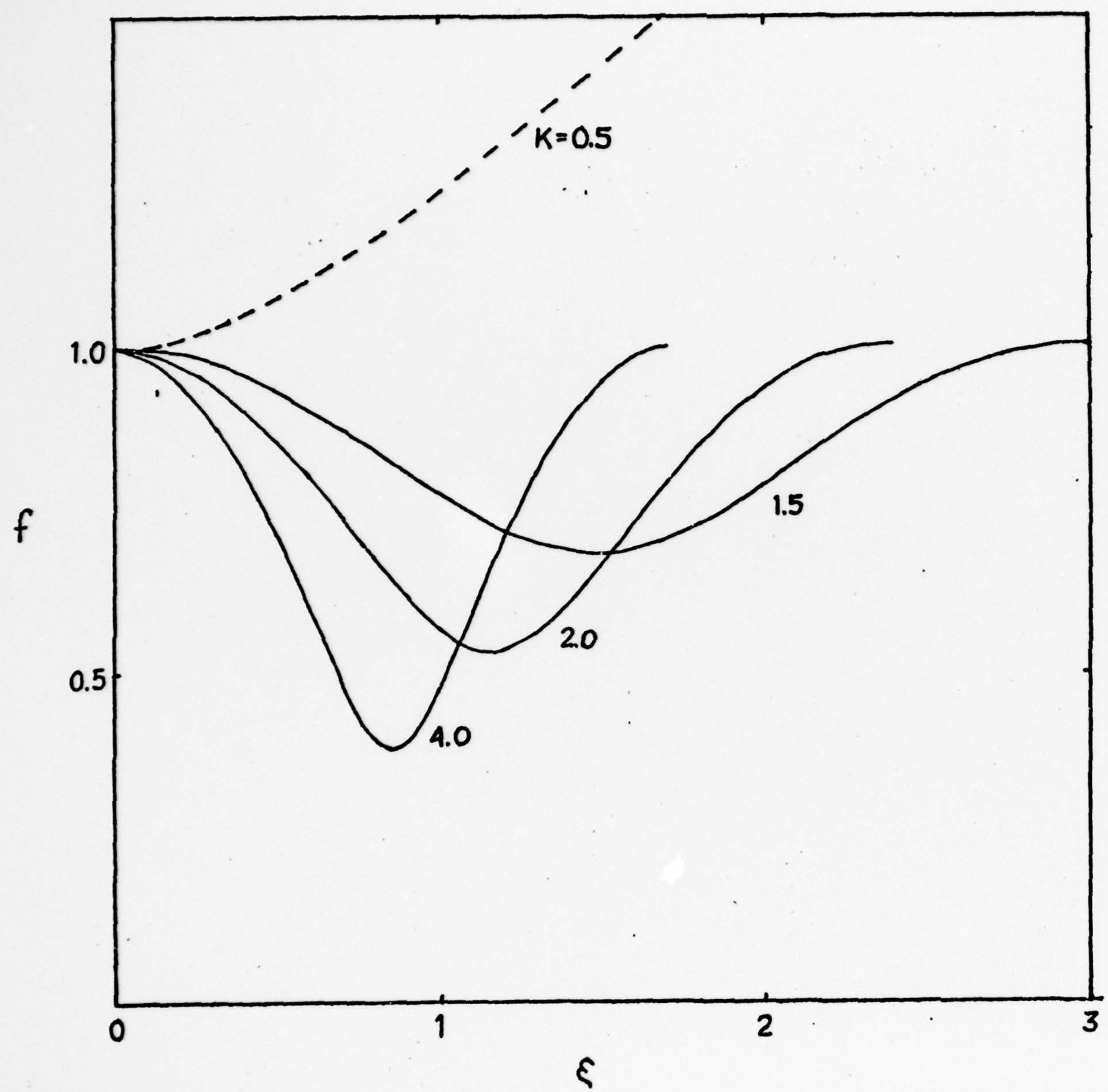

Fig. 3.

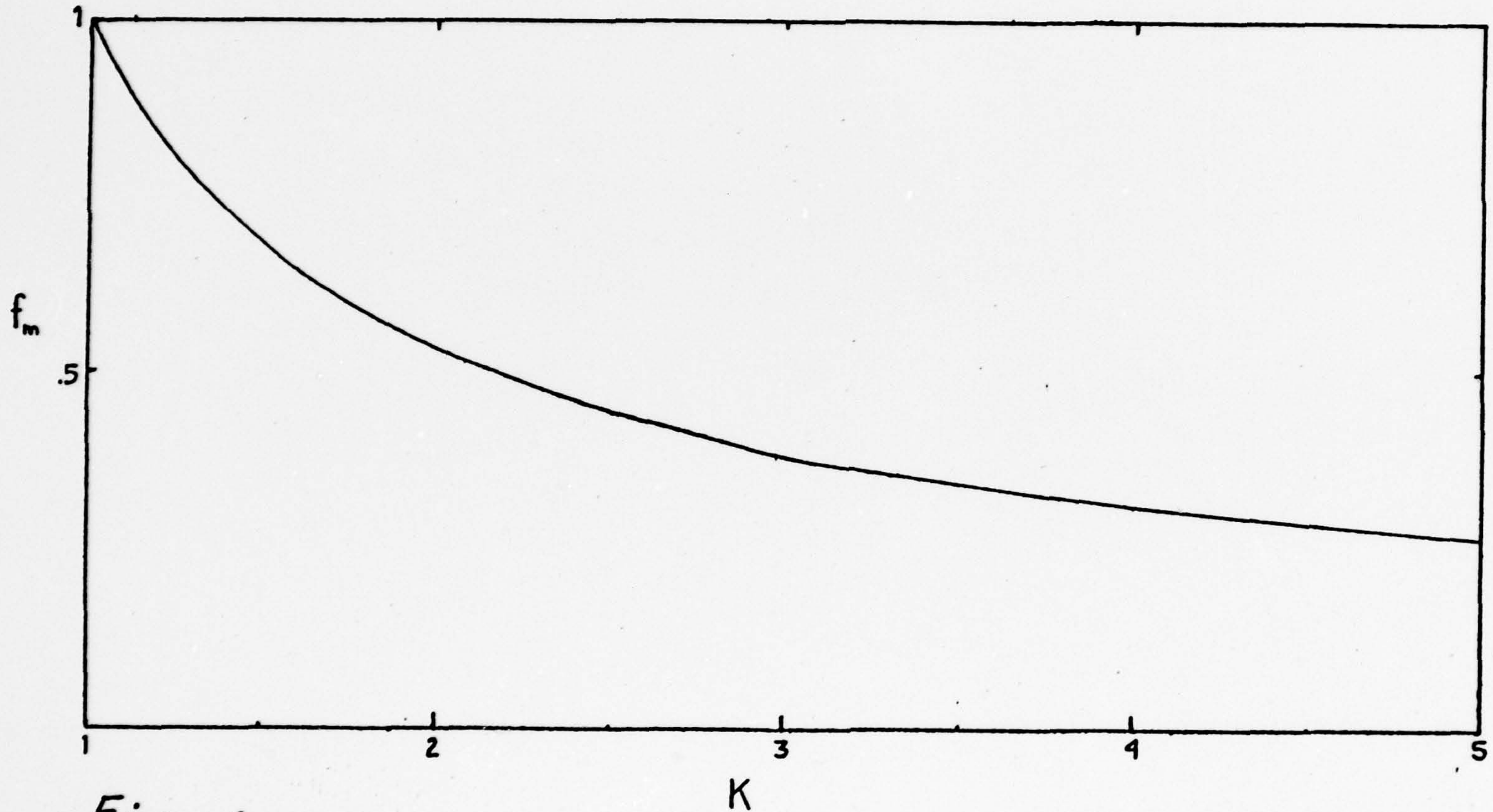

Fig. 4

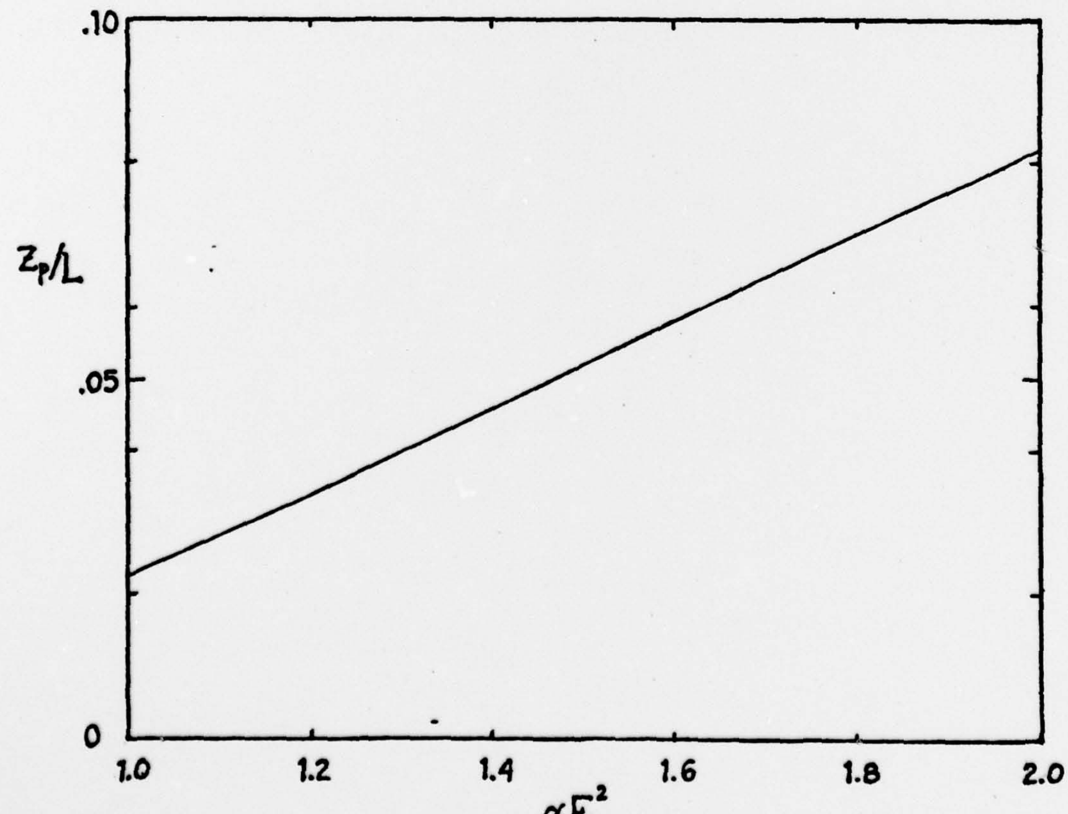

Fig. 5